Measurement of the total optical angular momentum transfer in optical tweezers

Simon Parkin, Gregor Knöner, Timo A. Nieminen, Norman R. Heckenberg
and Halina Rubinsztein-Dunlop

Centre for Biophotonics and Laser Science, School of Physical Sciences,
The University of Queensland, QLD 4072, Australia

Abstract

We describe a way to determine the total angular momentum, both spin and orbital, transferred to a particle trapped in optical tweezers. As an example an LG_{02} mode of a laser beam with varying degrees of circular polarisation is used to trap and rotate an elongated particle with a well defined geometry. The method successfully estimates the total optical torque applied to the particle. For this technique, there is no need to measure the viscous drag on the particle, as it is an optical measurement. Therefore, knowledge of the particle's size and shape, as well as the fluid's viscosity, is not required.

1   Introduction

The major application of optical tweezers, since their invention in 1986 [1], has been the study of microscopic biological systems. The first measurements were made on bacterial flagella and the transport of organelles within a cell [2, 3]. Advances in force measurement technology enabled high temporal and spatial resolution force measurements, for example, on motor proteins walking along microtubules [4, 5], polymerase transcribing DNA [6], protein folding [7, 8] and viruses packaging DNA [9]. Optical tweezers force measurements have provided a great insight into the physics of microscopic biological mechanisms and this application will continue due to the huge number of biological systems available for study. To further our understanding of the microscopic biological world, different quantitative measurement techniques need to be developed. Here we present a new technique to measure torques applied by optical tweezers opening a way to new quantitative studies of microscopic biological systems and their rotational dynamics.

Techniques to apply torques using optical tweezers by the transfer of angular momentum from the beam to the particle are well established. Absorption is the simplest transfer mechanism, whereby the particle absorbs the light's angular momentum. Both spin and orbital angular momentum have been transferred in this way [10, 11]. The first is due to the light's polarisation, while the orbital component is associated with the spatial distribution of the light's wavefront. A more elegant approach, that avoids unwanted heating due to absorption, is to trap a birefringent crystal with a circularly polarised beam [12]. This allows for efficient angular momentum transfer and the applied torque can be measured optically [13]. The advantage of an optical measurement of the torque is that knowledge of the properties of particle and its surrounding environment is not required. The particle's exact shape or the fluid's viscosity and refractive index do not affect the measurement of the applied torque. Based on this technique a micro-viscometer has been demonstrated using a spherical birefringent crystal as the object which was trapped and rotated [14]. However such crystals are somewhat difficult to produce and they are unstable in harsh media, such as acidic solutions. Therefore it would be useful to quantify torque transfer via orbital angular momentum as a larger torque efficiency is available [15] and suitable optically asymmetric objects are more abundant [16, 17]. Such objects include photopolymerised structures with sub-micron features which have been shown to function as light driven micromachines [18]. The ability to *optically* measure the torque applied to these micromachines would increase their functionality and would allow for quantitative measurements of rotational dynamics and feedback control.

Measurement of orbital angular momentum has been of interest in the field of quantum information and computing. Computer generated holograms have been used to measure the orbital angular momentum of single photons produced by parametric down-conversion. The experiments showed that orbital angular momentum is conserved during down conversion and that the orbital angular momentum states are entangled [19]. Measurement of the orbital angular momentum of an arbitrary beam using a similar technique has been demonstrated [20]. In these experiments the torque applied to a elongated phase object by a paraxial laser beam was determined by measuring the power in the forward scattered azimuthal modes. However to measure a large number of azimuthal modes requires a complicated setup and is impractical. This is the case for

measurements involving optical tweezers in particular, due to the variety of modes that are forward scattered by a particle trapped in a highly converging and diverging beam. Radial modes present a further complication for the measurement of the azimuthal modes.

Optical angular momentum can also be quantified by measuring the rotational Doppler shift introduced by rotating optical elements in the beam path [21]. The orbital component has been determined by measuring the frequency shift between two beams with different azimuthal indices [22]. In this experiment one of the beams is rotated using a 90° prism. In principle this technique could be used to measure the orbital angular momentum of an arbitrary beam. However it would be difficult to distinguish the different frequencies and, in particular, their amplitudes due to the noise from the rotating optical element required to give the frequency shift. A similar technique based on a Mach–Zehnder interferometer allows sorting of photons with different angular momentum states [23]. However, in practice this technique would also be difficult to implement in optical tweezers due to the series of interferometers required to measure the angular momentum of an arbitrary beam.

In this paper we present a method which enables the measurement of the total optical angular momentum transfer in optical tweezers. The spin component of the angular momentum can be measured purely optically as previously [17]. In order to measure the orbital component we take advantage of this existing method, and the linear relationship between the rotation rate of the trapped particle and the torque applied to it, to measure the orbital angular momentum transferred to the particle. To demonstrate this method we measure the torque applied to a trapped elongated particle with a well defined size and geometry. The accuracy of the particle's dimensions is important in order that the measured torque can be compared to the viscous drag on the particle as a test of the validity of the experimental method, but is not required for the torque measurement itself.

2   Theory

A convenient method to optically measure the torque applied to an object by the spin angular momentum of a laser beam was described in detail in [13]. The torque on the particle due to spin angular momentum transfer is given by $\tau_{\text{spin}} = \Delta\sigma P/\omega$ where $\Delta\sigma$ is the change in the degree of circular polarisation as the beam passes through the particle, P is the laser power and ω is the optical angular frequency.

For a particle rotating steadily the optically applied torque is equal to the drag torque applied by the surrounding liquid. In a low-Reynolds-number Newtonian fluid the drag torque is proportional to the rotation rate. The applied torque is also equal to the sum of the torques applied by the spin and orbital components. Therefore the torque acting on the trapped particle is given by:

$$\tau_{\text{total}} = \tau_{\text{orbital}} + \tau_{\text{spin}} = \Omega K \tag{1}$$

where K is an unknown constant of proportionality and Ω is the rotation rate. We assume that the the laser's frequency and power are known, which means τ_{spin} can be found directly from the change in polarisation ($\Delta\sigma$). This leaves an equation with two unknowns. By varying the independent variable (τ_{spin}) and measuring the rotation rate (Ω), K and the torque due to orbital angular momentum transfer (τ_{orbital}) can be determined.

3   Experiment

The optical tweezers setup (Fig. 1) was used to make measurements of the optical angular momentum transferred to trapped particles. A computer generated hologram [20] was used to generate a Laguerre–Gauss mode with an azimuthal index of two (LG_{02}) in the first diffraction order. The beam was expanded to fill the back aperture of an $100\times$ oil immersion objective (NA = 1.3) to yield the best trapping geometry and efficiency. The quarter wave plate before the objective was rotated to different angles to make left handed, right handed or linearly polarised laser light. An oil immersion condenser, with a numerical aperture greater than the objective (NA = 1.4), collected the diverging light from the optical trap created by the objective. A glass slide deflected a small percentage of the collected light to a photo-detector (photo-detector 3). The angle between the face of the glass slide and the axis of the laser beam's direction of propagation was as close to 90° as possible (the angle in the figure is exaggerated for clarity). This minimised both the amount of light deflected and the tendency for a certain polarisation to be deflected more strongly than its orthogonal counterpart. The spot size of the laser beam was greater than the area of the photo-detector so that the intensity of only a section of the beam was measured. The laser light transmitted through the glass slide was sent to a circular polarisation detection system. The spot size of the laser light incident on the two detectors (1 & 2) was smaller than the detector area so that the two detectors collect all the laser light collected by the condenser. The polarising beam splitter cube ensures that the two photo-detectors measure orthogonal linearly polarised components of the laser light. The quarter wave plate in front of the polarising beam splitter cube is aligned to ensure that right circularly
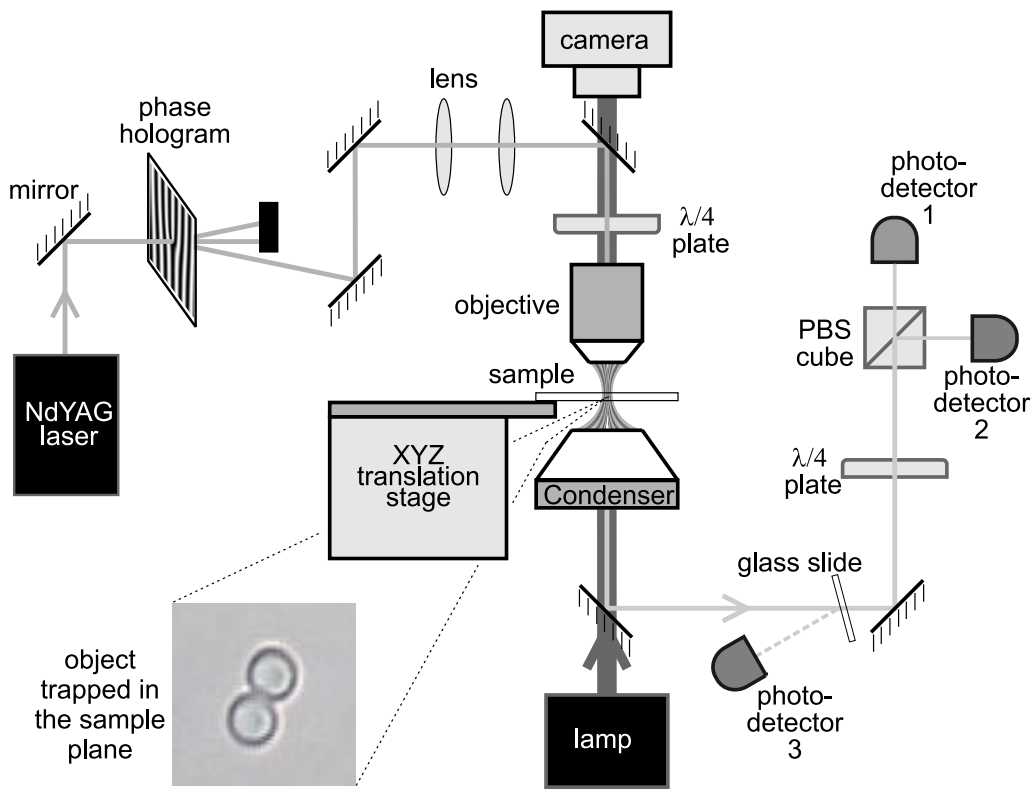
Figure 1: The optical tweezers setup used to make measurements of optical torque applied to a trapped object. The two polystyrene spheres that were trapped and rotated are shown in the inset. The phase hologram created a LG_{02} beam in the first diffraction order, which was used as the trapping beam. The three detectors measured the rotation rate of the particle and the change in polarisation of the trapping beam.

polarised light incident on the quarter wave plate is sent to one detector while left circularly polarised light is sent to the other detector.

A demonstration experiment was carried out in the optical tweezers with a simple asymmetric object that is readily available and has a relatively simple geometry. Two polystyrene beads (each two microns in diameter) were trapped and pushed together in the LG_{02} beam so that they behaved as one elongated object. Although in principle this object could be three dimensionally trapped, we chose to trap the beads two dimensionally against the glass slide to overcome the tendency for the elongated dimension of the object to align vertically in the trap.

Measurement of the laser power at the focus of the objective was needed for the determination of the optically applied torque on the trapped particle . A direct measurement is difficult due to the short working distance and high numerical aperture of the objective. Therefore the power at the focus was estimated by determining the transmission of the objective and condenser. As shown earlier, the optically applied torque also depends on the change in polarisation of the trapping beam. The circular polarisation detection system measures the power in each of the orthogonal circularly polarised components which allows the degree of circular polarisation to be found.

In order to determine the torque applied by the orbital component of the beam, the rotation rate of the trapped particles was measured. The fluctuations in intensity measured by photo-detector 3 corresponds to the rotation rate of the trapped particles. The particles have two fold symmetry which means that half the frequency of the intensity fluctuations is the particles' rotation rate. The constant of proportionality (K) between the rotation rate (Ω) and the optically applied torque (τ_{total}) can not be measured and was instead found by making three measurements of Ω and τ_{spin} at three different polarisations. Although two polarisations would be sufficient, linear, right handed circular and left handed circular polarisations were used.

4   Results and discussion

The change in polarisation of laser light that causes the rotation of two $2\,\mu$m beads, as well as their rotation rate, was measured for each of the three polarisations of the incident beam. A typical signal from the photo-detector that measures the particles' rotation rate is shown in Fig. 2(a). A sinusoidal fit to the signal shows that
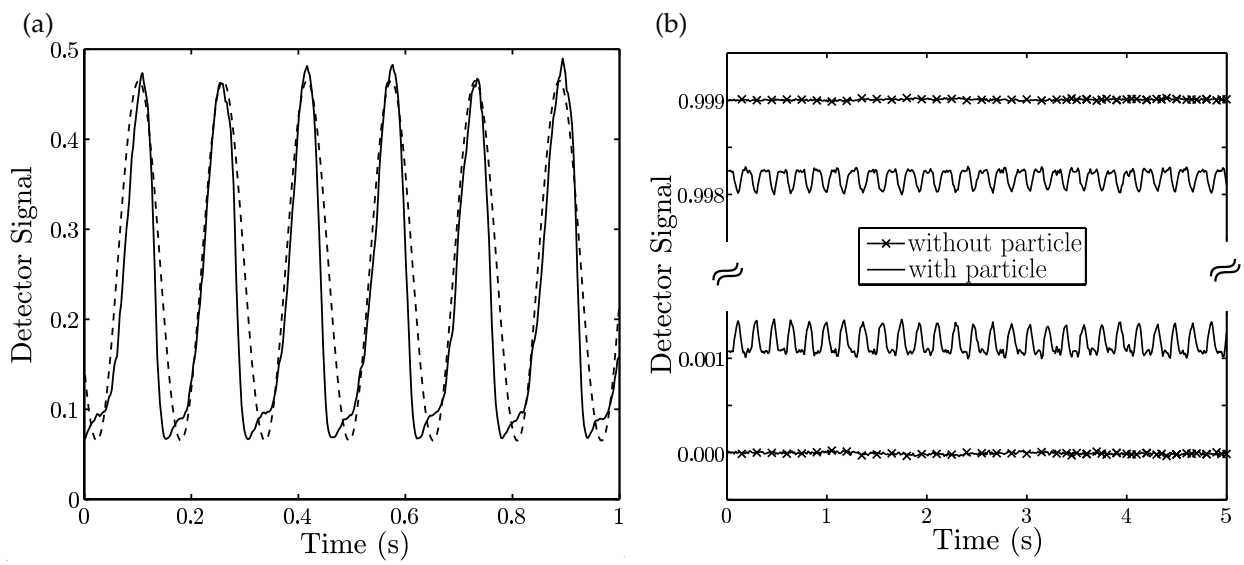
Figure 2: (a) Typical signal from photo-detector 3, from which the rotation rate of the particle was found. The solid line (—) is the actual signal from the detector and the dashed line (−−) is a sinusoidal fit of the data. (b) Typical signals from photo-detectors 1 and 2 for the case when no particle is trapped and also the case when the particles are trapped by the optical tweezers. The 'ripple' visible in the signal with the particle trapped is due to the rotation of the particle in the trap and has the same frequency as measured by photo-detector 3.

a constant rotation rate is maintained for a one second interval. The sampling duration was 5 seconds for each measurement and the average rotation rate over this interval was found.

The circularly polarised components of the beam were measured by 2 photodetectors. Typical signals from these detectors are shown in Fig. 2(b). The signals for each detector with and without a particle are plotted for the 5 second sampling interval. The average for each signal, over the time interval, was found. The degree of circular polarisation is calculated from these averages and the change in polarisation ($\Delta\sigma$) was found by subtracting the signal without a particle from the signal with a trapped particle.

The rotation rates measured for the three different polarisations are shown in Fig. 3. Each point on the plot has been averaged over 10 data sets. The torque transferred due to the spin component of the optical angular momentum (on the x axis of the plot) was calculated from the change in polarisation. The torque transfer via orbital angular momentum is found directly from this plot using equation 1 and setting τ_{spin} to zero:

$$\tau_{\text{orbital}} = K\Omega_0 = \frac{\Omega_0}{\text{slope}} = 0.017 \pm 0.003\hbar. \tag{2}$$

which means the torque due to the orbital component in this case is 5 times the torque due to the spin component. It should be noted here that this is an optical measurement of the torque and was made without knowledge of the particle's shape or refractive index, or the surrounding fluid's viscosity or refractive index. However, in this case, we do know the shape and size of the particle and the viscosity of the fluid, so the optical measurement can be compared to a viscous drag model.

For a steadily rotating particle the optically applied torque is equal to the viscous drag torque on the trapped particles. Therefore in order to check the measured value for the optically applied torque, we can model the viscous drag on the rotating particle. A simple model based on Stokes' drag on a translating sphere gives the following torque for two adjacent spheres rotating about their point of contact:

$$\tau_D = 12\pi\eta a^3\Omega \tag{3}$$

here a is the radius of each sphere (and the lever arm), η is the viscosity of the surrounding fluid and Ω is the rotation rate of the two spheres. The parameters used for this calculation are power (20 mW), rotation rate (2.4 Hz) and the individual spherical beads' radius (1 μm). The model gives a torque efficiency of 0.05 $\hbar$. We estimate the error in the model could be as much as 50 % due to wall effects and slipstreaming.

The results of the experiment and theory are of the same order of magnitude. This level of agreement is significant because it demonstrates that orbital torque can be estimated using this technique. The difficulty in the presented method is that a small change in polarisation signal needs to be measured in order to determine the orbital torque. For an orbital torque as small as 0.017$\hbar$, the signal measured by photo-detectors 1 and 2 only changed by 0.2%. This level of precision requires careful alignment of polarising optics and accurate reading
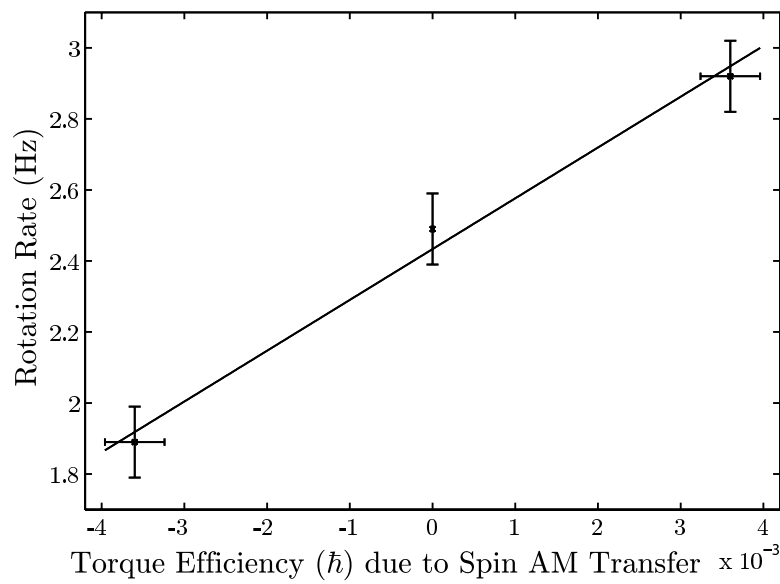
Figure 3: The rotation rate, Ω, of the trapped particle (two polystrene spheres) as a function of the optically applied torque due to the spin angular momentum of the trapping beam, τ_{spin}. The torque transfer of the orbital component is found from the slope $(1/K)$ and intercept (Ω_0) of the fit.

from the photodetectors. The ideal solution to this problem would be to boost the change in polarisation signal by choosing a particle that exhibits a stronger birefringence or form birefringence. Such a particle would allow for an accurate torque *measurement*. The method, as it stands, has only provided a good *estimate* of the optical torque transfer in optical tweezers. Fortunately techniques exist, such as photopolymerisation [18], which allow particles with sub-micron features to be fabricated. Such fine features can enhance the particles form birefringence which would make them very suitable for torque measurements.

The method described in this paper has the potential to make accurate measurements of the total optical angular momentum transfer in optical tweezers when there is negligible absorption. For proper application of the method a linear dependence of rotation rate on applied torque is assumed. This means the particle must freely rotate and that the behaviour of the fluid must be Newtonian. However, in the case of a particle that is not freely rotating, for example a particle attached to a cell, the calibration of the torque can be carried out prior to attaching the particle. Non-Newtonian behaviours, such as shear thinning are unlikely to affect the validity of the technique. Firstly because the shear rates created by particles in optical traps are usually too small for non-linearities to be observed [24] and secondly the trapping power can be reduced to decrease the rotation rate so that a linear regime of the fluid can be accessed. Once the torque transfer is calibrated then non-linearities could be studied.

Feasible applications for this technique are in studying microscopic biological systems and in the implementation of micromachines. For example previous studies on bacteria flagella [2, 25] could be extended by measuring the torque directly. Other applications include torsional elasticity measurements of single polymer chains or DNA strands as well as microviscosity measurements [18].

5   Conclusion

We have demonstrated a technique to measure both the spin and orbital components of the angular momentum transferred to a particle trapped in optical tweezers. For this technique, there is no need to measure the viscous drag on the particle, as it is an optical measurement. Therefore, knowledge of the particle's size and shape, as well as the fluid's viscosity, is not required. The technique successfully estimated the total optical torque on an asymmetric object when compared with a simple theoretical model. We have suggested that the accuracy of this method could be improved by using photo-polymerisation techniques to 'tailor-make' form birefringent particles. Such particles would have the advantage that both spin and orbital angular momentum could be more efficiently transferred from the trapping beam. In our experiment the orbital angular momentum was 5 times the magnitude of the spin component in this system which suggests that orbital angular momentum transfer will prove to be useful for quantitative study of torques in microscopic biological systems and in micromachine applications.

Acknowledgements

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